

## SHALLOW FOUNDATIONS

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### Summary

Shallow foundations are one of the most common types of foundations for buildings, retaining walls and other light structures. Spread footings are often the most inexpensive foundation solution compared with other types of foundation, such as piles and caissons. The basic principle in shallow foundation design is to provide a sufficiently large contact area between the footing and the soil so that loading imposed onto the footing can be supported by the underlying soil without failure and excessive settlement. The

calculation of the bearing capacity of a footing, which is the maximum bearing stress that the soil can take at failure, is dependent on the properties of the soil, such as soil cohesion and friction, size and depth of the footing, and surcharge at the ground surface. In conventional working stress designs, an appropriate factor of safety should be used, usually about 2.5 – 3, in calculating the allowable bearing capacity of the soil. In limit state designs, load and strength reduction factors are applied in determining the bearing capacity for the footing.

When calculating the bearing capacity of a footing based on the shear strength of the soil, there are basically two approaches: the total stress approach and the effective stress approach. The total stress approach is used to calculate the bearing capacity for the short term undrained condition of a saturated clay. The effective stress approach is used for the short and long term conditions for sand and the long term drained condition for clay.

Foundation settlement is often a governing factor in footing design. The amount of acceptable settlement for footings should be less than 25 mm. Depending on the type of soil, the bearing pressure must be limited to a value that will not result in more than 25 mm of total settlement over the life time of the structure.

Raft or mat foundations are a type of shallow foundation in which the entire building area is covered by the footing. A raft is used when the soil is weak or compressible and the total area of the individual footings is more than 50% of the building size. In designing a raft foundation, it is necessary to calculate the modulus of subgrade reaction. The modulus of subgrade reaction is defined as the ratio of the applied stress over the settlement of the raft foundation. It is used to calculate the loading on the raft for design purposes.

Construction of a footing requires excavation to a desired depth in the ground. The footing must be located below the maximum depth of frost penetration and seasonal variations of moisture, especially in expansive soils. During construction, ground water must be under control to prevent base instability or wetting and loosening of the soil at the base. Formwork is required unless individual shapes of the footing are excavated in the ground. The foundation of the footing should be examined by a qualified geotechnical engineer before pouring of the concrete to ensure that the soil is able to support the design bearing capacity.

## **1. Introduction**

### **1.1. Foundations for Buildings**

Shallow foundations are used to transfer loads from a building column or retaining wall to the soil near the ground surface. Shallow foundations are usually located within 2-5 m below the ground surface or basement level. Depending on the load bearing capacity of the soil and its compressibility characteristic, shallow foundations are to be located as close to the ground surface as possible. In regions of cold climates where the soil may freeze during winter months, the base of shallow foundations must be located below the maximum depth of frost penetration. This is to prevent settlement caused by freezing and thawing of the soil.

There are many types of shallow foundations, which range from simple spread footings to more complex combined footings. They all share one common principle, which is to distribute heavy load to a larger area that can be supported by the underlying soils without causing failure or excessive settlement.

## 1.2 Types of Shallow Foundations

The most common type of shallow foundation is a spread footing as shown in Figure 1. The spread footing is simply a concrete pad that spreads the column load to a larger surface area, thus reducing stresses on the soil. The pad is usually square or rectangular in shape, but can be round or other shapes, depending on the need and available space. Although normally, the column load is applied at the geometric center of the pad, it is not uncommon that the applied load may create eccentricity and tilting of the footing. In this case, it is important to limit the highest stress under the footing to be less than the bearing capacity of the soil.

A strip footing is normally used to support a wall. It is basically a long rectangular spread footing, see Figure 2. The loading on a strip footing is not necessarily located at its center because of the restriction on the property limits of a building. A strip footing is also called a continuous footing.

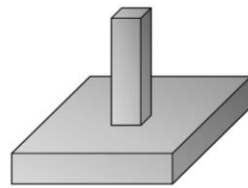


Figure 1a: Spread footing - square

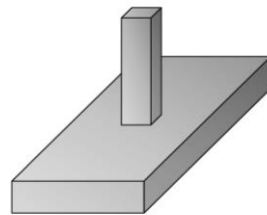


Figure 1b: Spread footing - rectangular

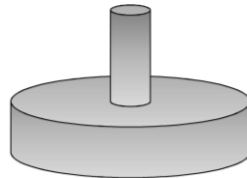


Figure 1c: Spread footing - circular

Figure 1. Spread footing

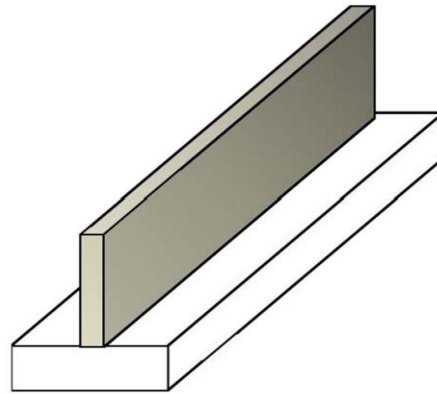


Figure 2. Strip or continuous footing

A combined footing supports two or more column loads, see Figure 3. The loading on each column is not necessary equal to each other and therefore, they can introduce moments in the footing. As a result, the stress distribution under a combined footing is not uniform.

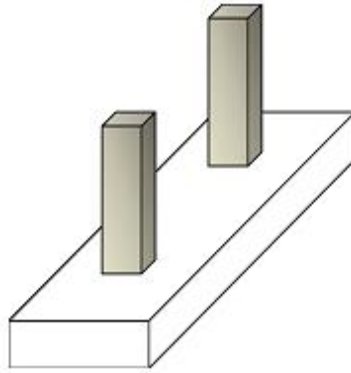


Figure 3. Combined footing

A strap footing is composed of two spread footings tied together with a grade beam, see Figure 4. A grade beam is used to transfer moments from one footing to the other. This provides added rotation stability to the footing which will otherwise fail due to the eccentricity of the loading.

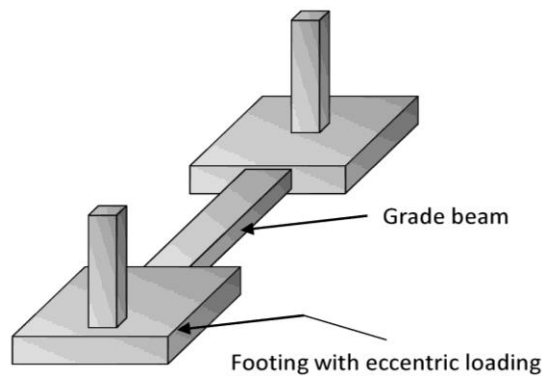


Figure 4. Strap footing

A raft or mat footing is used to support the entire building or entire loaded area by using one very large footing. Normally, the soil is quite soft and compressive in this case, and a large footing area is required to support the column loads. Therefore, the footing is so large that it basically covers the entire building area. When individual footings cover more than 50% of a building area, it becomes more economical to use a raft footing. A mat or raft footing is also used to support loading over a large area, such as a storage tank.

## 2. Bearing Capacity of Spread Footings

The calculation of the maximum stress that can be applied onto a spread footing is based on an assumed failure mode of the soil under the footing. This maximum pressure that causes failure is called the bearing capacity of the soil. Although the bearing capacity is dependent on the type of soil property, such as shear strength characteristics, it also depends on other factors that are not related to the properties of the material, such as the size of the footing. Unlike the shear strength of a soil, bearing capacity is not a property of the material.

### 2.1 Failure Modes of Spread Footings

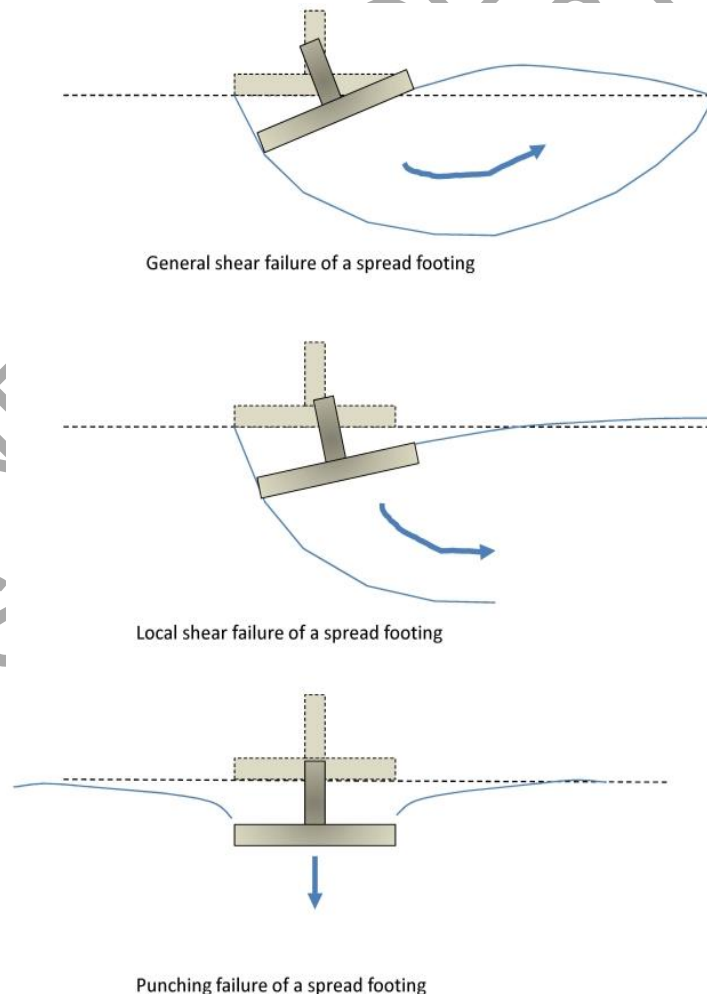


Figure 5. Various failure modes of a spread footing

In determining the bearing capacity for a footing, it is important to understand various modes of failure. There are basically three modes of failure of a spread footing: general, local and punching shear failures, see Figure 5.

### 2.1.1 General Shear Failure

General shear failure considers the failure of the soil under the footing which extends all the way to the ground surface. The failure zone can be divided into three regions: the rigid region directly beneath the footing, continuous shear region that rotates upward, and rigid shear region which extends to the ground surface. This is the most common mode of failure used in the analysis of a footing, see Figure 6.

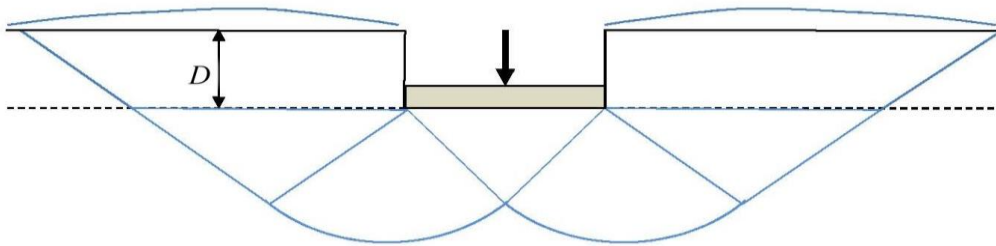


Figure 6. Theoretical failure zone under a spread footing

### 2.1.2 Local Shear Failure

Local shear failure involves a failure zone that does not extend to the ground surface, see Figure 5. The footing is either too deep below the ground surface or the soil is so compressible that it prevents the development of a complete failure surface that extends to the ground surface.

### 2.1.3 Punching Shear Failure

Punching shear failure occurs in a very compressible soil, such as peat. Shear surfaces cannot be developed due to the high compressibility of the material. The soil beneath the footing simply compresses and cannot sustain the load.

## 2.2 Ultimate Bearing Capacity Formula

In deriving the bearing capacity for a footing, the general shear failure mode is usually assumed. It is noted that this failure mode is not necessarily the actual failure mode of the footing, but the equation that it produces based on limit theorems gives the exact theoretical failure load. In other words, given the material parameters and failure mode, the equation gives the theoretical exact failure load that will cause failure of the footing.

Based on plasticity analysis and limit theorems, Terzaghi determined the following equation to calculate the maximum pressure that causes failure of strip footings:

$$q_u = CN_c + qN_q + \frac{1}{2}\gamma BN_\gamma \quad (1)$$

where  $N_c$ ,  $N_q$  and  $N_\gamma$  are the bearing capacity factors,

$q_u$  = ultimate bearing capacity of the footing;

$C$  = cohesion of the soil;

$q$  = surcharge at the base of the footing;

$\gamma$  = unit weight of the soil, and;

$B$  = width of the footing.

The bearing capacity factors  $N_c$ ,  $N_q$  and  $N_\gamma$  are given in Table 1. It is noted that the ultimate bearing capacity of the footing depends on the width  $B$  of the footing. That is why the bearing capacity is not a property of the soil.

$\phi$	$N_c$	$N_q$	$N_\gamma$	$\phi$	$N_c$	$N_q$	$N_\gamma$
10	8.3	2.5	0.4	31	32.7	20.6	17.7
11	8.8	2.7	0.5	32	35.5	23.2	20.8
12	9.3	3.0	0.6	33	38.6	26.1	24.4
13	9.8	3.3	0.8	34	42.2	29.4	28.8
14	10.4	3.6	1.0	35	46.1	33.3	33.9
15	11.0	3.9	1.2	36	50.6	37.8	40.1
15	11.6	4.3	1.4	37	55.6	42.9	47.4
17	12.3	4.8	1.7	38	61.4	48.9	56.2
18	13.1	5.3	2.1	39	67.9	56.0	66.8
19	13.9	5.8	2.5	40	75.3	64.2	79.5
20	14.8	6.4	2.9	41	83.9	73.9	95.1
21	15.8	7.1	3.5	42	93.7	85.4	114.0
22	16.9	7.8	4.1	43	105.1	99.0	137.1
23	18.0	8.7	4.9	44	118.4	115.3	165.6
24	19.3	9.6	5.7	45	133.9	134.9	200.8
25	20.7	10.7	6.8	46	152.1	158.5	244.6
26	22.3	11.9	7.9	47	173.6	187.2	299.5
27	23.9	13.2	9.3	48	199.3	222.3	368.7
28	25.8	14.7	10.9	49	229.9	265.5	456.4
29	27.9	16.4	12.8	50	266.9	319.1	568.6
30	30.1	18.4	15.1				

Table 1. Bearing Capacity Coefficients

The three terms on the right side of Eq. (1) represent the contribution of three components to the ultimate bearing capacity of the footing. The first term represents the contribution of the soil cohesion to the resistance due to shearing along the shear (or slip) surfaces as shown in Figure 6. Therefore, for a cohesionless material, such as sand, this term is zero.

The second term represents the contribution to the resistance from the surface loading above the base of the footing. If the footing is located at the ground surface and there is no loading besides the footing, this term is zero. In some cases, the footing is located

below the ground surface but this term is zero because failure of the footing does not involve failing of the material above the base, such as the case for the interior footing of a basement.

If the footing is buried into the ground, the surcharge can be determined based on the unit weight and depth of the material, i.e.,  $q = \gamma D$ , where  $D$  is the depth of the footing and  $\gamma$  is the unit weight of the surcharge, see Figure 6.

The third term represents contribution from the friction of the material. The bearing capacity factor  $N_\gamma$  is dependent on the friction of the soil. The amount of resistance depends on the size of the failure zone and therefore, this term depends on the width of the footing.

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### **Biographical Sketch**

**Dr. Dave Chan** is currently Associate Chair of Graduate Studies and Professor of the Department of Civil and Environmental Engineering at the University of Alberta in Canada. His expertise and research interest lie in computer analysis of soil deformation, constitutive modelling, liquefaction, landslide, debris flow and soil mobility analysis. Dr. Chan previously worked for Trow Consulting Engineers, a geotechnical consulting firm in Brampton, Ontario, as a senior engineer and subsequently worked as the head of geotechnical services in the Trow Oshawa office, and provided geotechnical consulting services. At the University of Alberta, he has also acted as a geotechnical consultant and numerical modeler in providing advice on numerical modeling for many projects that involve dams, slopes, excavations and foundations.

Dr. Chan is the author and co-author of many articles in refereed journals and conference proceedings. He is the associate editor of the *Canadian Geotechnical Journal*. He is a two time recipient of the R.M.Quigley award for the best paper in the *Canadian Geotechnical Journal*. He was one of the Canadian Geotechnical Conference colloquium lecturers on computer applications in geotechnical engineering. He has been invited to give lectures in Canada, Asia, China, Kazakhstan and Hong Kong. Dr. Chan's research is in the area of computer simulation of the deformation of soils and the development of constitutive models for deformation and mobility analysis of soils. He has developed finite and discrete element methods and computer codes for analyzing a variety of soil deformation problems.